Topic:- Simple Random Sampling With Replacement

Statement:-

Draw all possible samples of size 2 with replacement and the population is 2, 4, 6, 8, 10 and 12. Construct the sampling distribution of $\bar{x}$.

Verify the following

(i) $\mu_{\bar{x}} = \mu$

(ii) $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$

Solution:

$$\text{Total Samples} = (N)^n$$

$$= (6)^2$$

$$\text{Total Samples} = 36$$

S. No	Samples	$\bar{x}$	Samples	$\bar{x}$	Samples	$\bar{x}$
	2, 2	2	6, 2	4	10, 2	6
	2, 4	3	6, 4	5	10, 4	7
	2, 6	4	6, 6	6	10, 6	8
	2, 8	5	6, 8	7	10, 8	9
	2, 10	6	6, 10	8	10, 10	10
	2, 12	7	6, 12	9	10, 12	11
	4, 2	3	8, 2	5	12, 2	7
	4, 4	4	8, 4	6	12, 4	8
	4, 6	5	8, 6	7	12, 6	9
	4, 8	6	8, 8	8	12, 8	10
	4, 10	7	8, 10	9	12, 10	11
	4, 12	8	8, 12	10	12, 12	12

Sampling distribution of $\bar{x}$

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
2	1	1/36	2/36	4/36
3	2	2/36	6/36	18/36
4	3	3/36	12/36	48/36
5	4	4/36	20/36	100/36
6	5	5/36	30/36	180/36
7	6	6/36	42/36	294/36
8	5	5/36	40/36	320/36
9	4	4/36	36/36	324/36
10	3	3/36	30/36	300/36
11	2	2/36	22/36	242/36
12	1	1/36	12/36	144/36
	36	1	$\Sigma \bar{x}f(\bar{x}) = 252/36$	$\Sigma \bar{x}^2 f(\bar{x}) = 1974/36$

$$\mu_{\bar{x}} = \Sigma \bar{x} f(\bar{x}) = 252/36 = \boxed{7}$$

$$\sigma^2_{\bar{x}} = \Sigma \bar{x}^2 f(\bar{x}) - \{\Sigma \bar{x} f(\bar{x})\}^2$$

$$= \frac{1974}{36} - \{7\}^2 = \boxed{5.83}$$

Population Mean and Variance

$$\mu = \Sigma X/N = \frac{2+4+6+8+10+12}{6} = \boxed{7}$$

$$\sigma^2 = \Sigma X^2/N - \{\Sigma X/N\}^2 \Rightarrow \frac{364}{6} - (7)^2 \Rightarrow 11.66$$

verify that

(i) — $\mu_{\bar{x}} = \mu$

$7 = 7$

(ii) — $\sigma^2_{\bar{x}} = \sigma^2/n$ $11.66/2 = 5.83$

$5.83 =$

Topic :- Simple random Sampling without replacement

Statement :-

Draw all Possible Samples of size 3 without replacement and the Population is as ; 2, 4, 6, 8, 10, 12, 14, 16. Construct the Sampling distribution of $\bar{x}$.

Verify the followings

(i) $\mu_{\bar{x}} = \mu$

(ii) $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

Solution:

Total Samples can be drawn = $^8C_3 = \frac{8 \times 7 \times 6 \times 5!}{6!\ 3 \times 2!}$

Total Samples = 56

Samples	$\bar{x}$	Samples	$\bar{x}$	Samples	$\bar{x}$
2, 4, 6	4	⋮	⋮	⋮	⋮
2, 4, 8	4.67				
2, 4, 10	5.33				
⋮	⋮				

Sampling distribution of $\bar{x}$

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
4.00	1	1/56	4.00/56	16.00/56
4.67	⋮	⋮	⋮	⋮
5.33				
⋮				
14	1	1/56	14.00/56	196/56

$$\mu_{\bar{x}} = \Sigma \bar{x} f(\bar{x}) = \boxed{}$$

$$\sigma^2_{\bar{x}} = \Sigma \bar{x}^2 f(\bar{x}) - \{\Sigma \bar{x} f(\bar{x})\}^2 = \boxed{}$$

$$\mu = \Sigma X/N = \boxed{}$$

$$\sigma^2 = \Sigma X^2/N - \{\Sigma X/N\}^2 = \boxed{}$$

(i) $\mu_{\bar{x}} = \mu$

(ii) $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

Practical # 03

Statement:-

Draw all possible samples of size 3 without replacement and the population is 2, 4, 6, 8, 10, 12, 14, 16 and 18. Construct the sampling dist^n of $\bar{x}$.

Verify the followings

(i) — $\mu_{\bar{x}} = \mu$

(ii) — $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

Solution:-

We calculate the total samples with this formula can be used $^N c_n$.

N means total population observations

n Sample size

$$\text{Total Samples} = {}^N c_n = {}^9 c_3 = 84$$

Total samples can be drawn 84 then using the without replacement.

Without replacement:-

A ball can be selected in the basket. We cannot enter the ball again into basket is called without replacement samples concept.

	Samples	$\bar{x}$	Samples	$\bar{x}$	Samples	$\bar{x}$
1	2, 4, 6	4.00	4, 6, 8	6.00	6, 10, 16	10.67
2	2, 4, 8	4.67	4, 6, 10	6.67	6, 10, 18	11.33
3	2, 4, 10	5.33	4, 6, 12	7.33	6, 12, 14	10.67
4	2, 4, 12	6.00	4, 6, 14	8.00	6, 12, 16	11.33
5	2, 4, 14	6.67	4, 6, 16	8.67	6, 12, 18	12.00
6	2, 4, 16	7.33	4, 6, 18	9.33	6, 14, 16	12.00
7	2, 4, 18	8.00	4, 8, 10	7.33	6, 14, 18	12.67
8	2, 6, 8	5.33	4, 8, 12	8.00	6, 16, 18	13.33
9	2, 6, 10	6.00	4, 8, 14	8.67	8, 10, 12	10.00
10	2, 6, 12	6.67	4, 8, 16	9.33	8, 10, 14	10.67
11	2, 6, 14	7.33	4, 8, 18	10.00	8, 10, 16	11.33
12	2, 6, 16	8.00	4, 10, 12	8.67	8, 10, 18	12.00
13	2, 6, 18	8.67	4, 10, 14	9.33	8, 12, 14	11.33
14	2, 8, 10	6.67	4, 10, 16	10.00	8, 12, 16	12.00
15	2, 8, 12	7.33	4, 10, 18	10.67	8, 12, 18	12.67
16	2, 8, 14	8.00	4, 12, 14	10.00	8, 14, 16	12.67
17	2, 8, 16	8.67	4, 12, 16	10.67	8, 14, 18	13.33
18	2, 8, 18	9.33	4, 12, 18	11.33	8, 16, 18	14.00
19	2, 10, 12	8.00	4, 14, 16	11.33	10, 12, 14	12.00
20	2, 10, 14	8.67	4, 14, 18	12.00	10, 12, 16	12.67
21	2, 10, 16	9.33	4, 16, 18	12.67	10, 12, 18	13.33
22	2, 10, 18	10.00	6, 8, 10	8.00	10, 14, 16	13.33
23	2, 12, 14	9.33	6, 8, 12	8.67	10, 14, 18	14.00
24	2, 12, 16	10.00	6, 8, 14	9.33	10, 16, 18	14.67
25	2, 12, 18	10.67	6, 8, 16	10.00	12, 14, 16	14.00
26	2, 14, 16	10.67	6, 8, 18	10.67	12, 14, 18	14.67
27	2, 14, 18	11.33	6, 10, 12	9.33	12, 16, 18	15.33
28	2, 16, 18	12.00	6, 10, 14	10.00	14, 16, 18	16.00

Sampling distribution of $\bar{x}$

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
4.00	1	1/84	4.00/84	16.00/84
4.67	1	1/84	4.67/84	21.81/84
5.33	2	2/84	10.66/84	56.82/84
6.00	3	3/84	18.00/84	108.00/84
6.67	4	4/84	26.68/84	177.96/84
7.33	5	5/84	36.65/84	268.64/84
8.00	7	7/84	56.00/84	448.00/84
8.67	7	7/84	60.69/84	526.18/84
9.33	8	8/84	74.64/84	696.39/84
10.00	8	8/84	80.00/84	800.00/84
10.67	8	8/84	85.36/84	910.79/84
11.33	7	7/84	79.31/84	898.58/84
12.00	7	7/84	84.00/84	1008.00/84
12.67	5	5/84	63.35/84	802.64/84
13.33	4	4/84	53.32/84	710.76/84
14.00	3	3/84	42.00/84	588.00/84
14.67	2	2/84	29.34/84	430.42/84
15.33	1	1/84	15.33/84	235.01/84
16.00	1	1/84	16.00/84	256.00/84
	$\Sigma f = 84$	1	$\Sigma \bar{x}f(\bar{x}) = 840/84$	$\Sigma \bar{x}^2 f(\bar{x}) = 8960/84$

$$\mu_{\bar{x}} = \Sigma \bar{x} f(\bar{x}) = 840/84 = \boxed{10}$$

$$\sigma^2_{\bar{x}} = \Sigma \bar{x}^2 f(\bar{x}) - \{\Sigma \bar{x} f(\bar{x})\}^2$$

$$= 8960/84 - \{840/84\}^2 \Rightarrow 106.67 - 100 \Rightarrow \boxed{6.67}$$

Population Mean and Variance

$$\mu = \Sigma X / N \Rightarrow \quad \Sigma X = 2+4+6+8+10+12+14+16+18$$
$$= 90$$
$$N = 9$$

$$= \frac{90}{9}$$

$$\mu = 10$$

$$\sigma^2 = \Sigma X^2/N - \left\{\Sigma X/N\right\}^2$$

$$\Sigma X^2 = (2)^2+(4)^2+(6)^2+(8)^2+(10)^2+(12)^2+(14)^2+(16)^2+(18)^2$$
$$= 1140$$

$$= 1140/9 - \left\{90/9\right\}^2$$

$$\sigma^2 = 126.67 - (10)^2 \Rightarrow 26.67$$

Verify the Followings

(i) — $\mu_{\bar{x}} = \mu$

$$10 = \cancel{1}10$$

(ii) $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

$$6.67 = \frac{26.67}{3} \cdot \frac{(9-3)}{9-1}$$

$$6.67 = \frac{26.67}{3} \cdot (0.75)$$

$$6.67 = 6.67$$

Topic :- Sampling dist^n of Sample Proportion

Statement :-

A Population consists of $N=8$ values 1, 3, 4, 5, 6, 7, 8 and 10. Draw all Possible Samples of size 3 without replacement from the Population and find the Proportion of even numbers in the samples. Construct the sampling distribution of Sample Proportions and verify that

(i) $\mu_{\hat{p}} = P$

(ii) $Var(\hat{P}) = \frac{Pq}{n} \cdot \frac{N-n}{N-1}$

Solution :-

$$\text{Total Samples} = {}^{N}C_{n}$$

$$= {}^{8}C_{3}$$

$$= \frac{8!}{5!\,3!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!\,3 \times 2!}$$

$$\text{Total Samples} = 56$$

No.	Samples	Sample Proportion (P)	Samples	Sample Proportion (P)	Samples	Sample Proportion (P)
1	1, 3, 4	1/3	1, 7, 10	1/3	4, 5, 8	2/3
2	1, 3, 5	0	1, 8, 10	2/3	4, 5, 10	2/3
3	1, 3, 6	1/3	3, 4, 5	1/3	4, 6, 7	2/3
4	1, 3, 7	0	3, 4, 6	2/3	4, 6, 8	1
5	1, 3, 8	1/3	3, 4, 7	1/3	4, 6, 10	1
6	1, 3, 10	1/3	3, 4, 8	2/3	4, 7, 8	2/3
7	1, 4, 5	1/3	3, 4, 10	2/3	4, 7, 10	2/3
8	1, 4, 6	2/3	3, 5, 6	1/3	4, 8, 10	1
9	1, 4, 7	1/3	3, 5, 7	0	5, 6, 7	1/3
10	1, 4, 8	2/3	3, 5, 8	1/3	5, 6, 8	2/3
11	1, 4, 10	2/3	3, 5, 10	1/3	5, 6, 10	2/3
12	1, 5, 6	1/3	3, 6, 7	1/3	5, 7, 8	1/3
13	1, 5, 7	0	3, 6, 8	2/3	5, 7, 10	1/3
14	1, 5, 8	1/3	3, 6, 10	2/3	5, 8, 10	2/3
15	1, 5, 10	1/3	3, 7, 8	1/3	6, 7, 8	2/3
16	1, 6, 7	1/3	3, 7, 10	1/3	6, 7, 10	2/3
17	1, 6, 8	2/3	3, 8, 10	2/3	6, 8, 10	1
18	1, 6, 10	2/3	4, 5, 6	2/3	7, 8, 10	2/3
19	1, 7, 8	1/3	4, 5, 7	1/3		

Sampling distribution of Sample Proportion ($\hat{P}$)

$\hat{P}$	f	$f(\hat{P})$	$\hat{P} f(\hat{P})$	$\hat{P}^2 f(\hat{P})$	
0	4	4/56	0	0	
1/3	24	24/56	8/56	8/168	
2/3	24	24/56	16/56	32/168	
1	4	4/56	4/56	4/56	
Σ	56	1	28/56	52/168	

Now

$$\mu_{\hat{P}} = \Sigma \hat{P} f(\hat{P}) = \frac{28}{56} = \boxed{0.5} \text{ and}$$

$$\sigma^2_{\hat{P}} = \Sigma \hat{P}^2 f(\hat{P}) - \{\Sigma \hat{P} f(\hat{P})\}^2$$

$$= \frac{52}{168} - (0.5)^2 \Rightarrow 0.3095 - 0.25 = \boxed{0.0595}$$

Population Proportion P and the Population Variance pq

$P = X/N$, where X represents the number of even

$= 4/8 = \boxed{0.5}$ and

$\sigma^2 = pq \Rightarrow (0.5)(0.5) = 0.25$

$$Var(\hat{P}) = \frac{pq}{n} \cdot \frac{N-n}{N-1}$$

$$= \frac{(0.5)(0.5)}{3} \cdot \frac{(8-3)}{8-1}$$

$$= \frac{1.25}{21} \Rightarrow \boxed{0.0595}$$

Hence Proved.

Topic:- Sampling distribution of Sample Proportion

Statement:-

A population consists of $N=8$ values 1, 3, 4, 5, 6, 7, 8 and 10. Draw all possible samples of size 3 without replacement from the population and find the proportion of odd numbers in the samples. Construct the sampling distribution of sample proportions and verify that

(i) $\mu_{\hat{p}} = p$

(ii) $Var(\hat{p}) = \frac{pq}{n} \cdot \frac{N-n}{N-1}$

Solution:-

$$\text{Total Samples} = {}^{N}C_{n}$$

$$= {}^{8}C_{3}$$

$$= \frac{8!}{5!\,3!}$$

$$= \frac{8\times7\times6\times5!}{5!\;3\times2!}$$

$$\text{Total Samples} = 56$$

Sample No	Samples	Sample Proportion(P)	Samples	Sample Proportion(P)	Samples	Sample Proportion(P)
1	1, 3, 4	2/3	1, 7, 10	2/3	4, 5, 8	1/3
2	1, 3, 5	1	1, 8, 10	1/3	4, 5, 10	1/3
3	1, 3, 6	2/3	3, 4, 5	2/3	4, 6, 7	1/3
4	1, 3, 7	1	3, 4, 6	1/3	4, 6, 8	0
5	1, 3, 8	2/3	3, 4, 7	2/3	4, 6, 10	0
6	1, 3, 10	2/3	3, 4, 8	1/3	4, 7, 8	1/3
7	1, 4, 5	2/3	3, 4, 10	1/3	4, 7, 10	1/3
8	1, 4, 6	1/3	3, 5, 6	2/3	4, 8, 10	0
9	1, 4, 7	2/3	3, 5, 7	1	5, 6, 7	2/3
10	1, 4, 8	1/3	3, 5, 8	2/3	5, 6, 8	1/3
11	1, 4, 10	1/3	3, 5, 10	2/3	5, 6, 10	1/3
12	1, 5, 6	2/3	3, 6, 7	2/3	5, 7, 8	2/3
13	1, 5, 7	1	3, 6, 8	1/3	5, 7, 10	2/3
14	1, 5, 8	2/3	3, 6, 10	1/3	5, 8, 10	1/3
15	1, 5, 10	2/3	3, 7, 8	2/3	6, 7, 8	1/3
16	1, 6, 7	2/3	3, 7, 10	2/3	6, 7, 10	1/3
17	1, 6, 8	1/3	3, 8, 10	1/3	6, 8, 10	0
18	1, 6, 10	1/3	4, 5, 6	1/3	7, 8, 10	1/3
19	1, 7, 8	2/3	4, 5, 7	2/3		

Sampling distribution of Sample Proportion ($\hat{P}$)

$\hat{P}$	f	$f(\hat{P})$	$\hat{P}f(\hat{P})$	$\hat{P}^2 f(\hat{P})$
0	4	4/56	0	0
1/3	24	24/56	8/56	8/168
2/3	24	24/56	16/56	32/168
1	4	4/56	4/56	4/56
Σ	56	1	28/56	52/168

Now

$$\mu_{\hat{P}} = \Sigma \hat{P} f(\hat{P}) = \frac{28}{56} = \boxed{0.5} \text{ and}$$

$$\delta^2_{\hat{P}} = \Sigma \hat{P}^2 f(\hat{P}) - \{\Sigma \hat{P} f(\hat{P})\}^2$$

$$= \frac{52}{168} - (0.5)^2 = \boxed{0.0595}$$

Population Proportion P and the Population Variance pq

$P = X/N$, where X represents the Number of odd

$$= 4/8 = \boxed{0.5} \text{ and}$$

$$\delta^2 = pq \Rightarrow (0.5)(0.5) = 0.25$$

$$Var(\hat{P}) = \frac{pq}{n} \cdot \frac{N-n}{N-1}$$

$$= \frac{(0.5)(0.5)}{3} \cdot \frac{(8-3)}{8-1}$$

$$= \frac{1.25}{21} \Rightarrow \boxed{0.0595}$$

Topic:- Simple random Sampling

Statement:-

The Population is given as $N = 1, 3, 4, 5, 6$ with sample size of 2.

Prove that

(i) $E(s^2) = \sigma^2$

(ii) $E(s^2) = S^2$

Solution:-

First of all we solve the part (i)

$E(s^2) = \sigma^2$

Formula with replacement $= (N)^n$

Possible Samples $= (5)^2 = 25$

Samples	$\bar{x}$	$s^2 = \Sigma(x_i-\bar{x})^2/n-1$
1, 1	1.0	$(1-1)^2+(1-1)^2/2-1 = 0$
1, 3	2.0	$(1-2)^2+(3-2)^2/2-1 = 2$
1, 4	2.5	$(1-2.5)^2+(4-2.5)^2/2-1 = 4.5$
1, 5	3.0	
1, 6	3.5	
3, 1	2.0	
3, 3	3.0	
3, 4	3.5	
3, 5	4.0	
3, 6	4.5	
4, 1	2.5	
4, 3	3.5	
4, 4	4.0	
4, 5	4.5	
4, 6	5.0	
5, 1	3.0	
5, 3	4.0	
5, 4	4.5	
5, 5	5.0	
5, 6	5.5	
6, 1	3.5	
6, 3	4.5	
6, 4	5.0	
6, 5	5.5	$(6-5.5)^2+(5-5.5)^2/2.1 = 0.5$
6, 6	6.6	$(6-6.0)^2+(6-6.0)^2/2-1 = 0$

$s^2 = \Sigma(x_i-\bar{x})^2/n-1 = 74$

$s^2 = 74/25 \rightarrow$ Total Samples

$= 2.96$

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}$$

$$\bar{x} = \Sigma x / N = \frac{1+3+4+5+6}{5} = 3.8$$

$$= \frac{(1-3.8)^2 + (3-3.8)^2 + (4-3.8)^2 + (5-3.8)^2 + (6-3.8)^2}{5}$$

$$= \frac{14.8}{5}$$

$$= 2.96$$

Proved that

$E(s^2) = \sigma^2$

(ii) $E(s^2) = S^2$

without replacement $= ({}^{N}C_{n})$

Total Samples $= ({}^{5}C_{2}) = 10$

Samples	$\bar{y}$	$s^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2 / n-1$
1, 3	2.0	$(1-2.0)^2 + (3-2.0)^2 / 2-1 = 2.0$
1, 4	2.5	$(1-2.5)^2 + (4-2.5)^2 / 2-1 = 4.5$
1, 5	3.0	⋮
1, 6	3.5	⋮
3, 4	3.5	⋮
3, 5	4.0	⋮
3, 6	4.5	⋮
4, 5	4.5	⋮
4, 6	5.0	⋮
5, 6	5.5	$(5-5.5)^2 + (6-5.5)^2 / 2-1 = 0.5$
		$s^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2 / n-1 = 37$

$s^2 = 37/10$

$s^2 = 3.7$

$$S^2 = \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{N-1}$$

$$\bar{y} = \frac{\Sigma y}{N} = \frac{1+3+4+5+6}{5} = 3.8$$

$$S^2 = \frac{(1-3.8)^2 + (3-3.8)^2 + (4-3.8)^2 + (5-3.8)^2 + (6-3.8)^2}{5-1}$$

$$= \frac{14.8}{4}$$

$$S^2 = 3.7$$

We Proved that

$$E(s^2) = S^2$$

(i) $E(s^2) = \sigma^2$

$2.96 = 2.96$

(ii) $E(s^2) = S^2$

$3.70 = 3.70$

Topic :- Sampling Distribution of Differences b/w Means

Statement:-

Draw all possible random samples of size $n_1 = 2$ with replacement from finite population consisting of 4, 6, 8. Similarly draw all possible random samples of size $n = 2$ with replacement from another finite population consisting of 1, 2, 3.

(a) Find the possible differences between the sample means of the two populations.

(b) Construct the sampling distribution of $\bar{X}_1 - \bar{X}_2$ and compute its mean and variance.

(c) Verify that $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$ and $\sigma^2_{\bar{x}_1 - \bar{x}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

Possible Samples $(3)^2 = 9$

These two sets of samples and their means are given:

From Population 1			From Population 2		
Sample No	Sample Values	$\bar{x}_1$	Sample No	Sample Values	$\bar{x}_2$
1	4, 4	4	1	1, 1	1.0
2	4, 6	5	2	1, 2	1.5
3	4, 8	6	3	1, 3	2.0
4	6, 4	5	4	2, 1	1.5
5	6, 6	6	5	2, 2	2.0
6	6, 8	7	6	2, 3	2.5
7	8, 4	6	7	3, 1	2.0
8	8, 6	7	8	3, 2	2.5
9	8, 8	8	9	3, 3	3.0

Part -(a)

The 81 Possible differences $\bar{x}_1 - \bar{x}_2$ are Presented in the following table.

Differences of Independent Means

$\bar{x}_2$ \ $\bar{x}_1$	4	5	6	5	6	7	6	7	8
1.0	3.0	4.0	5.0	4.0	5.0	6.0	5.0	6.0	7.0
1.5	2.5	3.5	4.5	3.5	4.5	5.5	4.5	5.5	6.5
2.0	2.0	3.0	4.0	3.0	4.0	5.0	4.0	5.0	6.0
1.5	2.5	3.5	4.5	3.5	4.5	5.5	4.5	5.5	6.5
2.0	2.0	3.0	4.0	3.0	4.0	5.0	4.0	5.0	6.0
2.5	1.5	2.5	3.5	2.5	3.5	4.5	3.5	4.5	5.5
2.0	2.0	3.0	4.0	3.0	4.0	5.0	4.0	5.0	6.0
2.5	1.5	2.5	3.5	2.5	3.5	4.5	3.5	4.5	5.5
3.0	1.0	2.0	3.0	2.0	3.0	4.0	3.0	4.0	5.0

(b) The Sampling distribution of $\bar{X}_1 - \bar{X}_2$.

$\bar{x}_1 - \bar{x}_2$ $(=d)$	f	$f(\bar{x}_1 - \bar{x}_2)$	$d f(d)$	$d^2 f(d)$
1.0	1	1/81	1/81	1.0/81
1.5	2	2/81	3/81	4.5/81
2.0	5	5/81	10/81	20.0/81
2.5	6	6/81	15/81	37.5/81
3.0	10	10/81	30/81	90.0/81
3.5	10	10/81	35/81	122.5/81
4.0	13	13/81	52/81	208/81
4.5	10	10/81	45/81	202.5/81
5.0	10	10/81	50/81	250.0/81
5.5	6	6/81	33/81	181.5/81
6.0	5	5/81	30/81	180.0/81
6.5	2	2/81	13/81	84.5/81
7.0	1	1/81	7/81	49.0/81
Total	81	1	324/81	1431/81

$$\mu_{\bar{x}_1 - \bar{x}_2} = \Sigma(\bar{x}_1 - \bar{x}_2) f(\bar{x}_1 - \bar{x}_2)$$

$$= \Sigma d f(d) = \frac{324}{81} = \boxed{4} \text{ and}$$

$$\sigma^2_{\bar{x}_1 - \bar{x}_2} = \Sigma(d - \mu_{\bar{x}_1 - \bar{x}_2})^2 f(d) = \Sigma d^2 f(d) - [\Sigma d f(d)]^2$$

$$= \frac{1431}{81} - \left(\frac{324}{81}\right)^2 = \boxed{5/3}$$

Part – (c)

The mean and variance of the First Population are

$$\mu_1 = \frac{4+6+8}{3} = 6, \text{ and}$$

$$\sigma_1^2 = \frac{(4-6)^2 + (6-6)^2 + (8-6)^2}{3} = \frac{8}{3}$$

The mean and variance of the Second Population are

$$\mu_2 = \frac{1+2+3}{3} = 2, \text{ and}$$

$$\sigma_2^2 = \frac{(1-2)^2 + (2-2)^2 + (3-2)^2}{3} = \frac{2}{3}$$

Now

$$\mu_{\bar{x}_1 - \bar{x}_2} = 4$$

$$\mu_1 - \mu_2 = 6 - 2 = 4$$

$$\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{8}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2}$$

$$\frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$$

$$4 = 4$$

$$\sigma^2_{\bar{x}_1 - \bar{x}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$5/3 = 5/3$$

Proved that.

Revised the all Previous seven Practicals before the Midterm Exam.

Practical no: 8

Compute the Sample size in each stratum under Proportional allocation and Neyman allocation.

Farm Size	No. of Farms (N_h)	Avg. Corn Per Acres ($\bar{y}_h$)	St. Deviation (S_h)
0 — 40	394	5.4	8.3
41 — 80	461	16.3	13.3
81 — 120	391	24.3	15.3
121 — 160	334	34.5	19.3
161 — 200	169	42.1	24.3
201 — 240	113	50.1	26.0
241 & Above	148	63.8	35.2

$\boxed{N = 2010}$

Compare the precision of these methods in that of simple random sampling.

Solution: Proportional allocation

$N = 2010$, $n = 100$ $\quad n_h = n \cdot \frac{N_h}{N}$

$n_1 = 20$, $n_2 = 23$, $n_3 = 19$, $n_4 = 17$

$n_5 = 8$, $n_6 = 6$, $n_7 = 7 \Rightarrow \boxed{100}$

Neyman allocation

$$n_h = n \cdot \frac{N_h S_h}{\sum_{h=1}^{k} N_h S_h} \rightarrow = \cancel{34,118.10}$$

34084.3

$n_1 = 10$, $n_2 = 18$, $n_3 = 17$, $n_4 = 19$

$n_5 = 12$, $n_6 = 9$, $n_7 = 15$

Layman

Page # 2

Prop.

$N_h S_h^2 \quad N_h^2 S_h^2/n_h \quad N_h^2 S_h^2/n_h \quad N_h(\bar{y}_h - \bar{Y})^2 \quad (N-N_h)S_h^2$

⋮

685,838 139,55,820 116,47,178 556,439 60,83,139

$$\bar{Y} = \frac{1}{N}\sum_{h=1}^{k} N_h \bar{y}_h \Rightarrow \frac{52,885}{2010} \Rightarrow 26.31$$

Proportional Allocation

$$V(\bar{y}_{st})_{Prop.} = \frac{1}{N^2}\sum_{h=1}^{k} N_h^2 S_h^2/n_h - \frac{1}{N^2}\sum_{h=1}^{k} N_h S_h^2$$

$$= \frac{1}{(2010)^2}(139,55820) - \frac{1}{(2010)^2}(68,5838)$$

$$= 3.45 - 0.17$$

$$V(\bar{y}_{st})_{Prop.} = 3.28$$

Layman Allocation

$$V(\bar{y}_{st})_{lay} = \frac{1}{N^2}\sum_{h=1}^{k} N_h^2 S_h^2/n_h - \frac{1}{N^2}\sum_{h=1}^{k} N_h S_h^2$$

$$= 2.88 - 0.17$$

$$V(\bar{y}_{st})_{Lay} = 2.71$$

$\frac{100}{2010} = 0.0498$

Simple random Sampling

$$V(\bar{y}_{ran}) = V(\bar{y}_{st})_{Prop.} + \frac{1-f}{n(N-1)}\left[\sum_{h=1}^{k} N_h(\bar{y}_h - \bar{Y})^2 - \frac{1}{N}\sum_{h=1}^{k}(N-N_h)S_h^2\right]$$

$$= 3.28 + \frac{0.9502}{200900}(553,413)$$

$$V(\bar{y}_{ran}) = 3.28 + 2.62 \Rightarrow 5.90$$

① Relative Efficiency of Proportional allocation as Compared to SRS. Page # ③

$$V(\bar{y}_{ran}) / V(\bar{y}_{st})_{Prop} \times 100$$

$$5.90 / 3.28 \times 100 \Rightarrow 179.88$$

②

$$V(\bar{y}_{ran}) / V(\bar{y}_{st})_{Ney} \times 100$$

$$5.90 / 2.71 \times 100 \Rightarrow 217.71$$

Practical no: 9

In the following data showing the Population of 69 cities divided into 4 strata the Number of cities in each stratum the average Population and standard deviation of the Population are given in the following table $N = 69$

Stratum	N_h	$\bar{y}_h$	S_h
1	30	135	26
2	20	218	62
3	12	318	53
4	7	566	155

(i) Proportional allocation
(ii) Equal Allocation
(iii) Optimum Allocation

Allocate the Sample of size 20 to estimate the total Population of cities by three methods. and check the efficiency of the above method of allocation among themselves Particularly with simple random sampling.

Solution:

Proportional allocation

$$n_h = n \cdot \frac{N_h}{N}$$

$n_1 = 20 \cdot 30/69 = 9$; $n_2 = 20 \cdot 20/69 = 6$

$n_3 = 20 \cdot 12/69 = 3$; $n_4 = 20 \cdot 7/69 = 2$

Equal allocation:

$$n_1 + n_2 + n_3 + n_4 = n$$

$$5 + 5 + 5 + 5 = 20$$

Layman allocation:-

$$n_h = n \cdot \frac{N_h S_h}{\sum_h^k N_h S_h}$$

$$n_1 = \frac{20 \times (30 \times 26)}{(30)(26) + (20)(62) + (12)(53) + (7)(155)} = \frac{15,600}{3741} = 4$$

$$n_2 = \frac{20 \times (20 \times 62)}{3741} = 7$$

$$n_3 = 20 \times (12 \times 53)/3741 = 3$$

$$n_4 = 20 \times (7 \times 155)/3741 = \frac{6}{20}$$

Proportional Allocation

$N_1^2 S_1^2/n_1 = (30)^2(26)^2/9 = 67,600.00$

$N_2^2 S_2^2/n_2 = (20)^2(62)^2/6 = 256,266.67$

$N_3^2 S_3^2/n_3 = (12)^2(53)^2/3 = 134,832.00$

$N_4^2 S_4^2/n_4 = (7)^2(155)^2/2 = 588,612.50$

$10,47,311.17$

Equal Allocation:

$N_1^2 S_1^2/n_1 = (30)^2(26)^2/5 = 121,680.00$

$N_2^2 S_2^2/n_2 = (20)^2(62)^2/5 = 307,520.00$

$N_3^2 S_3^2/n_3 = (12)^2(53)^2/5 = 80,899.20$

$N_4^2 S_4^2/n_4 = (7)^2(155)^2/5 = 235,445.00$

$745,544.2$

Optimum Allocation

$N_1^2 S_1^2/n_1 = (30)^2(26)^2/4 = 152,100.00$

$N_2^2 S_2^2/n_2 = (20)^2(62)^2/7 = 219,657.14$

$N_3^2 S_3^2/n_3 = (12)^2(53)^2/3 = 134,832.00$

$N_4^2 S_4^2/n_4 = (7)^2(155)^2/6 = 196,204.17$

$702,793.31$

Page # 3

$$v(\hat{y}_{ran}) = v(\bar{y}_{st_{Pop}}) + N^2 \sum_{h=1}^{K} \frac{N_h(\bar{y}_{hi} - \bar{y})^2}{n \cdot N}$$

$$v(\bar{y}_{st_{Pop}}) = \frac{1}{N^2}\left[N_1(N_1-n_1)\frac{S_1^2}{n_1} + N_2(N_2-n_2)\frac{S_2^2}{n_2} + N_3(N_3-n_3)\frac{S_3^2}{n_3} + N_4(N_4-n_4)\frac{S_4^2}{n_4}\right]$$

$$= 1/(69)^2\left[30(30-9)\frac{(26)^2}{9} + 20(20-6)\frac{(62)^2}{6} + 12(12-3)\frac{(53)^2}{3} + 7(7-2)\frac{(155)^2}{2}\right]$$

$$= 1/4761\left[47,320 + 179,386.67 + 101,124 + 420,437.5\right]$$

$$= 1/4761\,(748,268.17)$$

$$v(\bar{y}_{st_{Pop}}) = 157.166$$

$$\bar{y} = \frac{N_1\bar{y}_1 + N_2\bar{y}_2 + N_3\bar{y}_3 + N_4\bar{y}_4}{N}$$

$$= \frac{(30)(135) + (20)(218) + (12)(318) + (7)(566)}{69}$$

$$\bar{y} = 16188/69 \Rightarrow 234.609$$

$$N_h(\bar{y}_h - \bar{y})^2$$

$$30(135 - 234.609)^2 \Rightarrow 297,658.6$$

$$20(218 - 234.609)^2 \Rightarrow 5517.2$$

$$12(318 - 234.609)^2 \Rightarrow 83,448.7$$

$$7(566 - 234.609)^2 \Rightarrow 768740.0$$

$$11,55,364.5$$

$$V(\hat{y}_{ran}) = V(\bar{y}_{st})_{Prop.} + \frac{N^2 \sum_{h=1}^{k} N_h (N_h - \bar{Y})^2}{n \cdot N}$$

$$= 157.166 + \frac{(69)^2 \times 11,55,364.5}{(20)(69)}$$

$$= 157.166 + 39,86,007.3$$

$$V(\hat{y}_{ran}) = 39,86,164.5$$

① $V(\hat{y}_{ran}) / V(\hat{y}_{st})_{Prop.} \times 100$

$39,86,164.5 / 10,47,311.17 \times 100 \Rightarrow 380.609$

② $V(\hat{y}_{ran}) / V(\hat{y}_{st})_{Eq.} \times 100$

$39,86,164.5 / 745,544.2 \times 100 \Rightarrow 534.665$

③ $V(\hat{y}_{ran}) / V(\hat{y}_{st})_{Opt./Neyman} \times 100$

$39,86,164.5 / 702,793.3 \times 100 \Rightarrow 567.2$

Practical # 10

Topic:- Stratified random Sampling with SRS

Statement:-

The following table shows the 1920 and the 1930 a number of inhabitants in thousand of 64 large cities in USA. The cities are arranged in two strata the first containing the 16 larger cities and the second the remaining 48 cities.

First - stratum:- For 1920 Population

797	773	748	734	588	577	507
507	457	438	415	401	387	381
324	315					

Second - stratum:- For 1920 Population

314	298	296	258	256	243	238	237
235	235	216	208	206	192	180	179
172	172	163	162	161	159	153	144
138	138	138	138	136	132	130	126
121	120	119	118	118	116	116	113
113	110	110	108	106	104	101	100

First-stratum:- For 1930 Population

900	827	781	805	670	1238	573	634
578	487	442	451	459	464	400	366

Second-stratum:- For 1930 Population

364	317	328	302	288	291	253	291
308	272	284	255	270	214	195	260
209	183	163	253	232	260	201	147
292	164	143	169	139	170	150	143
113	115	123	154	140	119	130	127
100	107	114	111	163	116	122	134

→ Find the standard error of the estimated total

→ By Simple rand Sampling.

→ By stratified random sampling with Proportional allocation.

→ Stratified random Sampling with equal allocation

Solution:

	1920 Total and Sum of squares		1930 Total and Sum of squares	
Stratum	$\Sigma(x_{hi})$	$\Sigma(x_{hi})^2$	$\Sigma(y_{hi})$	$\Sigma(y_{hi})^2$
1	8349	47,56,619	10,070	71,45,450
2	7941	14,74,871	9496	21,41,720
	16,290	62,31,490		

For the Complete Population in 1920, we find

$$X = x_1 + x_2$$

$$X = 8349 + 7941 = 16,290$$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{16290}{64} = 254.53$$

$$S^2 = \frac{1}{N-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{N}\right]$$

$$= \frac{1}{64-1}\left[62,31,490 - \frac{(16,290)^2}{64}\right]$$

$$S^2 = 33098$$

(i) For Simple random Sampling

$$V(\hat{y}_{ran}) = \frac{N^2 S^2}{n} - \frac{N-n}{N}$$

$$= \frac{(64)^2(33098)}{24} - \frac{64-24}{64}$$

$$V(\hat{y}_{ran}) = 35,30,453$$

$$S.E(\hat{y}_{ran}) = 1879$$

(ii) For the Individual strata the variance are calculated as:

$$S_{hi}^2 = \frac{\Sigma(x_{hi} - \bar{x}_h)^2}{n_h - 1}$$

For first stratum, the variance is

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$= \frac{1}{n_1 - 1}\left[\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}\right]$$

$$= \frac{1}{16-1}\left[47,56,619 - \frac{(8349)^2}{16}\right]$$

$$S_1^2 = 26,667$$

And for second stratum, the variance is

$$S_2^2 = \frac{1}{n_2 - 1}\left[\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}\right]$$

$$= \frac{1}{48-1}\left[14,74,871 - \frac{(7941)^2}{48}\right]$$

$$S_2^2 = 3428$$

In Proportional allocation, we have

$$n_1 = 6 \quad , \quad n_2 = 18$$

$$V(\hat{y}_{Prop.}) = \frac{N-n}{n} \cdot (\Sigma N_h S_h^2)$$

$$= \frac{64-24}{24}\left[N_1 S_1^2 + N_2 S_2^2\right]$$

$$= \frac{40}{24}\left[16(26,667) + 48(3428)\right]$$

$V(\hat{y}_{Prop}) = 985,360$

$S.E(\hat{y}_{Prop}) = 993$

For equal Population, we have

$n_1 = 12 \quad , \quad n_2 = 12$

$$V(\hat{y}_{equal}) = \Sigma N_h (N_h - n_h) \frac{S_h^2}{n_h}$$

$$V(\hat{y}_{equal}) = N_1(N_1 - n_1) \cdot \frac{S_1^2}{n_1} + N_2(N_2 - n_2) \cdot \frac{S_2^2}{n_2}$$

$$= 16(16-12) \cdot \frac{26,667}{12} + 48(48-12) \cdot \frac{3428}{12}$$

$V(\hat{y}_{equal}) = 63,5856$

$S.E(\hat{y}_{equal}) = 797$

In this example equal sample size in the two strata are more precise than proportional allocation both are superior to simple random sampling.

For the Complete Population in 1930, we find

$$Y = y_1 + y_2$$

$$= 10,070 + 9498 = 19,568$$

$$\bar{Y} = \Sigma Y/N = \frac{19,568}{64} = 305.75$$

$$S^2 = \frac{1}{N-1}\left[\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}\right]$$

$$= \frac{1}{64-1}\left[92,87,170 - \frac{(19,568)^2}{64}\right]$$

$$S^2 = 52,448$$

(i) For Simple random Sampling

$$V(\hat{y}_{ran}) = \frac{N^2 S^2}{n} \cdot \frac{N-n}{N}$$

$$= \frac{(64)^2 (52,448)}{24} \cdot \frac{64-24}{64}$$

$$V(\hat{y}_{ran}) = 55,94,453$$

$$S.E(\hat{y}_{ran}) = 2365$$

(ii) For the Individual strata the variances are calculated as:

$$S_{hi}^2 = \frac{\Sigma (y_{hi} - \bar{y}_h)^2}{n_h - 1}$$

For the first stratum, the variance is

$$S_1^2 = \frac{1}{n_1 - 1}\left[\Sigma y_1^2 - \frac{(\Sigma y_1)^2}{n_1}\right]$$

$$= \frac{1}{16-1}\left[71,45,450 - \frac{(10,070)^2}{16}\right]$$

$$S_1^2 = 53,843$$

and the Second stratum, the variance is

$$S_2^2 = \frac{1}{n_2-1}\left[\Sigma y_2^2 - \frac{(\Sigma y_2)^2}{n_2}\right]$$

$$= \frac{1}{48-1}\left[21,41,720 - \frac{(9498)^2}{48}\right]$$

$$S_2^2 = 5581$$

Note that the stratum with the largest cities has a variance nearly 10 times that of the other stratum.

In Proportional allocation

$$n_1 = 6 \quad , \quad n_2 = 18$$

$$V(\hat{y}_{Prop.}) = \frac{N-n}{n}\left[\Sigma N_h S_h^2\right]$$

$$= \frac{64-24}{24} \cdot \left[16(53,843) + 48(5581)\right]$$

$$V(\hat{y})_{Prop.} = 18,82,293$$

$$S.E(\hat{y})_{Prop.} = 1372$$

For equal allocation, we have

$$n_1 = 12 \quad , \quad n_2 = 12$$

$$V(\hat{y}_{equal}) = \Sigma N_h(N_h - n_h) \cdot \frac{S_h^2}{n_h}$$

$$= 16(16-12) \cdot \frac{53,843}{12} + 48(48-12) \cdot \frac{5581}{12}$$

$$V(\hat{y})_{equal} = 10,90,827$$

$$S.E(\hat{y})_{equal} = 1044$$

Topic:- Ratio Method of Estimation

Statement:-

The values of Y_i and X_i in Population are as Follows;

$$Y_i = 3, 5, 6, 7, 8, 13$$
$$X_i = 1, 2, 2, 3, 3, 4$$

By Computing $\hat{R}$ For all Possible Samples of size 3 without replacement.

Show that

$$E(\hat{R}) = R$$

Solution:

$$\text{Total Samples} = {}^{6}C_{3} = 20$$

$$E(\hat{R}) = \Sigma y / \Sigma x = \frac{420}{150} = 2.8$$

$$R = \frac{\Sigma Y}{\Sigma X} = \frac{42}{15} = 2.8$$

$$E(\hat{R}) = R$$

$$2.80 = 2.80$$

Samples (y)	Sum (y)	Samples	Sum (x)
3, 5, 6	14	1, 2, 2	5
3, 5, 7	15	1, 2, 3	6
3, 5, 8	16	1, 2, 3	6
3, 5, 13	21	1, 2, 4	7
3, 6, 7	16	1, 2, 3	6
3, 6, 8	17	1, 2, 3	6
3, 6 13	22	1, 2, 4	7
3, 7, 8	18	1, 3, 3	7
3, 7, 13	23	1, 3, 4	8
3, 8, 13	24	1, 3, 4	8
5, 6, 7	18	2, 2, 3	7
5, 6, 8	19	2, 2, 3	7
5, 6, 13	24	2, 2, 4	8
5, 7, 8	20	2, 3, 3	8
5, 7, 13	25	2, 3, 4	9
5, 8, 13	26	2, 3, 4	9
6, 7, 8	21	2, 3, 3	8
6, 7, 13	26	2, 3, 4	9
6, 8, 13	27	2, 3, 4	9
7, 8, 13	28	3, 3, 4	10
	$\Sigma y = 420$		$\Sigma x = 150$

Practical # 12 Lecture # 13 Paper no II

Topic:- Ratio method of Estimation and Regression Estimation

Statement:-

The values of Y_i and x_i in Population are as follows

$$Y_i = 3, 5, 6, 7, 8, 13, 15, 17$$

$$X_i = 1, 2, 2, 3, 3, 3, 4, 5$$

(i) Check that regression Y on X is straight line Passing through the origin.

(ii) By Computing $\hat{R}$ for all Possible simple random sample of size 2 without replacement.

Solution:— (i) $\Sigma X_i = 23$, $\Sigma Y = 74$, $\Sigma XY = 253$

$$\beta = \frac{\Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{n}}{\Sigma Y^2 - \frac{(\Sigma Y)^2}{n}}$$

$$\alpha = \bar{Y} - \beta \bar{X}$$

To fit regression line

$$Y = -1.3875 + 3.70 X$$

Part — (ii)

Samples (Y)	Sum (Y)	Samples (X)	Sum X
3, 5	8	1, 2	3
3, 6	9	1, 2	3
3, 7	10	1, 3	4
⋮	⋮	⋮	⋮

$$E(\hat{R}) = \Sigma y / \Sigma x = 518/161 = 3.22$$

$$R = \Sigma Y / \Sigma X = 74/23 = 3.22$$

$$E(\hat{R}) = R$$

$$3.22 = 3.22$$

Proved that

Topic:- Ratio method of Estimation

Statement :-

The values of Y_i and X_i in Population are as follows;

$$Y_i = 2, 3, 4, 5, 6, 6, 7, 9$$

$$X_i = 1, 1, 2, 3, 3, 4, 5, 5$$

By Computing $\hat{R}$ for all Possible Samples of size 3 without replacement show that

$$E(\hat{R}) = R$$

Solution:-

$$\text{Total Samples} = {}^{8}C_{3} = \frac{8\times7\times6\times5!}{5!\,3\times2!}$$

$$\text{Total Samples} = 56$$

Samples (y)	Sum (y)	Samples (x)	Sum (x)	y/x
2, 3, 4	9	1, 1, 2	4	2.25
2, 3, 5	10	1, 1, 3	5	2.00
2, 3, 6	11	1, 1, 3	5	2.20
2, 3, 7	12	1, 1, 4	6	2.00
⋮	⋮	⋮	⋮	⋮

$$E(\hat{R}) = \frac{\Sigma(y/x)}{m} = \frac{98}{56} = 1.75$$

m = Total samples

$$R = \frac{\Sigma Y}{\Sigma X} = \frac{42}{24} = 1.75$$

$$E(\hat{R}) = R$$

$$1.75 = 1.75$$

Proved that

Statement:-

Family No	Size (x_1)	Income (x_2)	Food Cost (y)
1	2	62	14.3
2	3	62	20.8
3	3	87	22.7
4	5	65	30.5
5	4	58	41.2
6	7	92	28.2
7	2	88	24.2
8	4	79	30.0
9	2	83	24.2
10	5	62	30.2
11	3	63	13.4
12	6	62	19.8
13	4	60	29.4
14	4	75	27.1
15	2	90	22.2
16	5	75	37.7
17	3	69	22.6
18	4	83	36.0
19	2	85	20.6
20	4	73	27.7
21	2	66	25.9

Family No.	Size (x_1)	Income (x_2)	Food Cost (y)
22	5	58	23.3
23	3	77	39.8
24	4	69	16.8
25	7	65	37.4
26	3	77	34.8
27	3	69	28.7
28	6	95	63.0
29	2	77	19.5
30	2	69	21.6
31	6	69	18.2
32	4	67	20.1
33	2	63	20.7
	119		907

Above the table shows No. of Person (x_1) the weekly Family Income (x_2) and the weekly expenditure on Food (y) in a simple random sample of (33) low Income Family.

→ Estimate Average weekly expenses on Food Per family.

→ Average weekly expenses on food Per Person.

→ The Percentage of the Income spend on Food. Compute the standard error of these estimate.

(a) — $\bar{y} = \Sigma y_i / n$

$$S.E(\bar{y}) = \frac{1}{\sqrt{n}} \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}}$$

(b) — $\hat{R}_1 = \frac{\Sigma y_i}{\Sigma x_i}$

$$S.E(\hat{R}_1) = \frac{1}{\bar{x}^2\sqrt{n}} \sqrt{\frac{\Sigma(y_i - Rx_1)^2}{n-1}} \quad \text{(Ignoring fpc)}$$

$$R_1 = \frac{1}{\bar{x}^2\sqrt{n}} \sqrt{\Sigma y_1^2 + R_1^2 \Sigma x_1^2 - 2\hat{R}_1 \Sigma y_i x_1}$$

$\Sigma x_1^2 = 533$; $\Sigma x_2^2 = 177{,}250$

$\Sigma y^2 = 28{,}266.28$; $\Sigma x_1 y = 3594.5$

$\Sigma x_2 y = 66{,}665.5$

<u>Solution</u>:

Part (a)

$$\bar{y} = \frac{\Sigma y}{n} = \frac{907}{33} = 27.485$$

$$S.E(\bar{y}) = \frac{1}{\sqrt{n}} \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}}$$

$$= \frac{1}{\sqrt{n(n-1)}} \sqrt{\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}}$$

$$= \frac{1}{\sqrt{33(33-1)}} \sqrt{28{,}266.28 - \frac{(907)^2}{33}}$$

$$S.E(\bar{y}) = \boxed{1.762}$$

Part — (b)

$$\hat{R}_1 = \frac{\Sigma y_i}{\Sigma x_i}$$

$$= \frac{907}{119} = 7.622 \text{ Per Person}$$

$$\bar{x}_1 = \frac{\Sigma x_1}{n}$$

$$= \frac{119}{33} = 3.606 \text{ Per Person}$$

$$S.E(\hat{R}_1) = \frac{1}{\bar{X}_1\sqrt{n}} \sqrt{\Sigma y^2 + \hat{R}_1^2 \Sigma x_1^2 - 2\hat{R}_1 \Sigma y x_1 / n-1}$$

$$= \frac{1}{3.606\sqrt{33}} \sqrt{\frac{28206.28 + (58.095)(533) - 2(7.622)(3594.5)}{33-1}}$$

$$= \frac{1}{20.716} (11.694)$$

$$S.E(\hat{R}_1) = \boxed{0.565}$$

$$\hat{R}_2 = \frac{\Sigma y_i}{\Sigma x_2}$$

$$= \frac{907}{2394}$$

$$\bar{X}_2 = \frac{\Sigma x_2}{n} = \frac{2394}{33} = 72.545$$

$$S.E(\hat{R}_2) = \frac{1}{\bar{X}_2\sqrt{n}} \sqrt{\frac{\Sigma y_i^2 + \hat{R}_2^2 \Sigma x_2^2 - 2\hat{R}_2 \Sigma x_2 y}{n-1}}$$

$$= \frac{1}{72.545\sqrt{33}} \sqrt{\frac{28206.28 + (0.379)^2(171256) - 2(0.379)(66665.5)}{33-1}}$$

$$S.E(\hat{R}_2) = 6.70$$

Revised all the 16 Practicals
and discussed if any Query.